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How the Classical Pointer Moves

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Hepp proposed to solve the measurement problem by taking as pointer observable one of the classical observables arising naturally in infinite quantum systems. Here a time evolution is proposed which finishes the measurement in finite time. This time evolution arises naturally as the limit of measurement interactions of finite systems when the particle number tends to infinity.

1. INTRODUCTION

Following the pioneering work of Hepp (1972), the use of classical pointer observables in quantum measurements has been extensively[2] studied. Which problems are solved by the introduction of a classical pointer observable, which are not solved, and which arise newly?

Let, for simplicity, the state space of the observed system 0 be two-dimensional, $\mathbf{C}^2$. Take as measured observable the spin σ_0^3 in 3-direction. Denote by $\psi_\pm$ the eigenstates, and by $P_\pm$ the projectors onto them. As apparatus A take an infinite chain of spins numbered 1, 2, . . . , Then the Hilbert space of the joint system 0 plus apparatus is $\mathcal{H} := \bigotimes_{k=0}^{\infty} \mathbf{C}_k^2$, where $\bigotimes_{k=0}^{\infty}$ denotes the complete infinite tensor product as defined by von Neumann (1936). $\mathcal{H}$ is a nonseparable Hilbert space.

In finite systems one can define the algebra of observable to be the C^*-algebra or von Neumann algebra generated by the one-particle observables. This yields all observables of the finite system. Let us make the same definition for our infinite system: take as algebra of observables the C^*-algebra

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[2] See, e.g., Bóna (1973), Frigerio (1974), Whitten-Wolf and Emch (1976), Machida and Namiki (1980), Araki (1980, 1986), Kudaka *et al.* (1989), Namiki and Pascazio (1991), Landsman (1991), and Nakazato and Pascazio (1992). For philosophical discussions of the classical pointer observable in quantum measurements see Bub (1988, 1989), Landsman (1995), and Robinson (1990, 1994).

$\mathcal{A}$ generated by the one-particle observables. $\mathcal{A}$ is called the algebra of quasilocal observables. In contradistinction to the finite case, $\mathcal{A}$ now does not contain all bounded or all compact operators on the Hilbert space. The elements of $\mathcal{A}$ are those bounded operators which can be approximated in norm by operators with only finitely many entries differing from the identity operator.

Denote by $\mathcal{A}''$ the weak closure of $\mathcal{A}$ in $\mathcal{B}(\mathcal{H})$. It can be regarded as an extended algebra of observables. $\mathcal{A}''$ is a von Neumann algebra with nontrivial center $\mathcal{Z}(\mathcal{A}'')$: there are some nontrivial observables in $\mathcal{A}''$ which commute with all other observables. Such observables are called *classical.* This is the case, for example, for the average spin of the infinite chain. The attribute "classical" is justified for such observables: they have a dispersion-free expectation values in all pure states.

Two states in which some classical observable has a different value are *disjoint.* They belong to subspaces of $\mathcal{H}$ which are not connected by any observable. These subspaces are the superselection sectors. The transtion probabilities between states in different sectors vanish. No observable transformation can carry a state into a disjoint one. In other words: no quantum mechanical interference terms exist between disjoint states. A vector in $\mathcal{H}$ which is the sum of disjoint vectors has the same expectation value in all observables as the mixture of the vector states.

Also, any mixed state allows an ignorance interpretation with respect to all classical observables. Every pure state is an eigenstate of all classical observables. Thus every decomposition of a mixed state into pure states is a decomposition into states in which all classical observables have dispersion-free values. Therefore it is justified to say that the pointer observable in each single experiment has a well-defined, but perhaps unknown value. Another reason to take a classical pointer is that only measurement results obtained with a classical pointer are robust under small perturbations of the final state (Breuer *et al.*, 1993).

2. THE PROBLEM OF TIME EVOLUTION

Traditionally the time evolution in quantum mechanics is generated by a self-adjoint operator, the Hamiltonian H. In the Heisenberg picture the time evolution of an observable A is given by

$$U_i A U_i^* =: \alpha_i(A)$$

where $U_t = \exp(itH)$. If H is an observable, U does not change the value of the classical observables.

The maps α_t: $\mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ are automorphisms of the von Neumann algebra $\mathcal{B}(\mathcal{H})$. Generalizing the concept of Hamiltonian time evolution, one can take time evolutions to be any pointwise norm-continuous one-parameter automorphism group of the C^*-algebra of observables. This definition in two ways generalizes Hamiltonian evolutions: First, it may happen that a given one-parameter group of automorphisms does not allow a strongly continuous unitary implementation. Second, even if the automorphisms are unitarily implementable in some representations, the Hamiltonian may depend on the representation. But on the other hand, there are unitary time evolutions which are not automorphic: if the Hamiltionian is not an observable, the algebra of observables is not necessarily mapped onto itself.

Automorphic time evolutions have difficulties in describing the evolution of a classical pointer. This was pointed out by Hepp (1972, Lemma 2). He showed that if two states ρ_1,ρ_2 on a C^*-algebra are in the same superselection sector, and if α_t is an automorphism, then $\rho_1 \circ \alpha_t$ and $\rho_2 \circ \alpha_t$ will again be in the same sector, although possibly in a different one from where they started. Applied to measurements, this has the following consequence: The initial states of the joint system are all in the superselection sector corresponding to pointer "ready"; if the time evolution is described by a one-parameter automophism group of the C^*-algebra, then for every finite time the evolved states will again be in the same sector. Therefore every classical observable, in particular the pointer observable, will agree on the evolved states.

There are two ways to circumvent Hepp's no-go theorem. First, and this way of escape was emphasized by Hepp, one can take the limit $t \to \infty$. Even if ρ_1, ρ_2 are in the same sector and the time evolution is described by a norm-continuous automorphism group α_1, $\rho_1 \circ \alpha_t$ and $\rho_2 \circ \alpha_t$ can converge for $t \to \infty$ to disjoint states. Second, one can consider nonautomorphic time evolutions.

For two reasons there are problems with first way of escape. If one insists that the experimentally relevant pointer observable really is classical, then it can change its value just in the infinite-time limit. Real measurements, however, only take finite time. Furthermore, convergence to the disjoint final states ρ_1^∞, ρ_2^∞ only takes place in the weak*-topology on the state space. This means that for any *fixed* set of observables $A_1, \ldots, A_n'$ we have

$$\lim_{t\to\infty} (\rho_i(\alpha_i(A_i)) - \rho_1^\infty(A_i)) = 0$$

and similarly for ρ_2. But for any given time and any given $\in$ it is possible to find an observable $B(t)$ for which the interference terms between $\rho_i \circ \alpha_t$ and $\rho_2 \circ \alpha_t$ are bigger than $\in$ [The operator $B(t)$ can simply be chosen in such a way that the Heisenberg time evolution is undone.] It was for this reason that Bell doubted the physical significance of Hepp's result.

3. A NONAUTOMORPHIC TIME EVOLUTION

Usually nonautomorphic time evolutions emerge for open systems interacting with an environment. In this final section I would like to discuss the possibility of nonautomorphic time evolutions for *closed* systems. This evolution will take the initial state in finite time into a state with pointer value depending on the value of the measured observable in the initial state. It closely resembles Hepp's (1972) big-bang time evolution or the evolutions proposed by Wan (1980) and Bub (1988, 1989), although at least the latter two did not seem to realize that this kind of evolution carries observable operators into unobservable ones. The point I want to make is that this evolution arises as the infinite-particle limit of standard measurement interactions.

As finite-volume Hamiltonian take

$$H_N := \sigma_0^3 \bigotimes_{k=1}^{N} \sigma_k^1 = P_+ \bigotimes_{k=1}^{N} \sigma_k^1 - P_- \bigotimes_{k=1}^{N} \sigma_k^1$$

The resulting time evolution

$$U_N(t) := \exp(itH_N) = \exp(it\sigma_0^3) \bigotimes_{k=1}^{N} \exp(it\sigma_k^1)$$

turns the first N spins of an initial state $\Psi_\pm := \psi_\pm \bigotimes_{k=1}^{\infty} \phi_k$ in positive or negative direction depending on the spin state of the measured system and leaves the rest of the chain unchanged,

$$U_N(t)\Psi_\pm = \psi_\pm \bigotimes_{k=1}^{N} \exp(\pm it\sigma_k^1)\phi_k \bigotimes_{k=N+1}^{\infty} \phi_k$$

If the pointer observable were the average spin of the first N spins H_N would be the natural interaction coupling the measured observable σ_0^3 to the generators of pointer displacement. It is the direct analogue of von Neumann's (1932, p. 236) unitary measurement interaction.

Let us now take the limit $N \to \infty$ of the finite-volume dynamics,

$$H_\infty := \sigma_0^3 \bigotimes_{k=1}^{\infty} \sigma_k^1$$

and as pointer observable the average spin of the infinite chain. $U_\infty(t)$ carries an initial state $\Psi_\pm$ into

$$U_\infty(t)\Psi_\pm = \psi_\pm \bigotimes_{k=1}^{\infty} \exp(\pm it\sigma_k^1)\phi_k$$

The whole infinite chain of spins is turned in a direction depending on the spin state of the measured system. The value of the pointer changes immediately. Since the average spin is classical, the evolved states $U_\infty(t)\Psi_\pm$ are disjoint immediately.

The evolution proposed seems to solve the measurement problem. A superposed initial state $(c_+ \psi_+ + c_-\psi_-) \bigotimes_{k=1}^{\infty} \mathbf{ø}_k$ is carried into $c_+U_\infty(t)\Psi_+ + c_-U_\infty(t)\Psi_-$. Although this is still a vector in $\mathcal{H}$, it represents a mixed state since for all observables in $\mathcal{A}$ it has the same expectation values as the mixed state

$$|c_+|^2|c_+U_\infty(t)\Psi_+\rangle\langle c_+U_\infty(t)\Psi_+| + |c_-|^2|c_-U_\infty(t)\Psi_-\rangle\langle c_-U_\infty(t)\Psi_-|$$

Every decomposition of this mixed state into pure states is a decomposition into eigenstates of the pointer observable because every pure state is an eigenstate of the pointer observable. Thus an ignorance interpretation of this mixed state is justified.

But the time evolution $U_\infty \cdot U_\infty^*$ is not an automorphism of $\mathcal{A}$ or of any weak closure of it in $\mathcal{H}$. For example, the observable $|\psi_+\rangle\langle\psi_-| \bigotimes_{k=1}^{\infty} \mathbf{1}$ is carried into $|\psi_+\rangle\langle\psi_-| \bigotimes_{k=1}^{\infty} \exp(-it\sigma_k^1)$ which is not in $\mathcal{A}$ or any weak closure of it. Since $U_\infty \cdot U_\infty^*$ is not an automorphism Hepp's no-go theorem does not apply and indeed U_∞ carries the initial states $\Psi_\pm$, which are in the same superselection sector, into disjoint states.

In which sense can we say that U_N converges to U_∞? In the weak operator topology $U_N(t)$ does not converge to $U_\infty(t)$ because $\langle U_N\Psi_\pm|U_\infty\Psi_\pm\rangle = 0$ for all N, whereas $\langle U_\infty\Psi_\pm|U_\infty\Psi_\pm\rangle = 1$. Neither does U_N converge to U_∞ in the strong operator topology since $\|U_N\Psi_\pm - U_\infty\Psi_\pm\| = 2$ for all N. But the weak*-limit of the states $U_N\Psi_\pm$ exists and equals $U_\infty\Psi_\pm$: for any fixed local operator A (for which only the first, say, l entries are different from the identity operator) $\langle U_N\Psi_\pm|A|U_N\Psi_\pm\rangle$ is independent of N for $N > l$.

Nonautomorphic time evolutions also arise in some lattice models with long-range interactions, for examples, in the mean-field models of Hepp and Lieb (1973), Morchio and Strocchi (1987), Bóna (1988), or Unnerstall (1990). These models do not admit a time evolution described by a one-parameter group of automorphisms of $\mathcal{A}$. Therefore Landsman (1991) suggested that the kind of dynamics arising in the presence of long-range interactions might be used to describe measurement evolutions. But this does not work. If the algebra of observables is extended to be the closure $\mathcal{M}$ of $\mathcal{A}$ in some suitable weak topology, the evolution is an automorphism of the extended algebra $\mathcal{M}$. In general $\mathcal{M}$ will contain classical observables, and their value will change in the course of time. But Hepp's Lemma 2 can be applied also to $\mathcal{M}$ and yields the result that states which initially agree on all classical observables will always do so. The present time evolution is different because it is not an automorphism of any von Neumann algebra $\mathcal{M}$, $\mathcal{A} \subset \mathcal{M} \subset \mathcal{A}''$.

Is a nonautomorphic time evolution really a bona fide time evolution? It seems very strange that it carries some observable operator into an unobservable one. Still, I think that we should consider seriously the possibility that U_∞ is realized in nature. After all, there are many realistic interactions where the time evolution fails to be an automorphism. This is the case, for example, for any attractive interaction if we take as observables sums of products of smeared field operators (Narnhofer and Thirring, 1990).

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